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The energy equations solutions for black hole accretion flows with the outflow: Description the cooling mechanism

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Elhadj Becherrair Belghitar [a,*], Salah Tlili [b], Zahida Malki [a], Yacine Marif [b]

[a] Department of Physics, Faculty of Mathematics & Matter Sciences, University of Ouargla, LRPPS Laboratory, 30000 Ouargla, Algeria

[b] Department of Physics, Faculty of Mathematics & Matter Sciences, University of Ouargla, LENREZA Laboratory, 30000 Ouargla, Algeria

Email : tlilisalah2007@gmail.com

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Abstract

While accretion disks are forming around the black hole,the energy resulting from this phenomenon is partlytransformed into the disk's kinetic energy, with the remainder converted into thermal energy. Winds and outflows play a crucial role in transferring mass and angular momentum, thereby affecting the thermal energy of the disk considerably. A large fraction of the mass lost through the wind can constitute a substantial component of the overall mass transfer rate. The principal aim of this research is to investigate the thermal equilibrium configurations of an accretion disk surrounding a black hole including the effects of outflows.However, the secondary objective of this work is to study the influence of outflows on the temperature behavior of accreting matter around a black hole, considering both optically thin and optically thick accretion disks. The analytical models developed in this work to study the black hole accretionshow strong with earlier numerical simulation results.Our results show that the outflow is of great importance in the regions where radiative cooling is more dominant, and this influence is apparent in both SSD and SLE solutions.

Keywords: Black hole, Thermal energy, Outflow, Thermal equilibrium, Temperature,Radiative cooling.

1. Introduction

Many objects in the universe are the source of vast quantities of energy; certain double stars emitting X-rays and specific galactic nuclei are objects of this type. Matter accretion onto black holes is one of the most powerful mechanisms for energy production in the universe.Studying the behavior of accretion disks around black hole is fundamental to understanding how galaxies and their black holes evolve together. To understand the mechanisms governing to the growth of these structures, we adopt a physical modal composed of a black hole and surroundingaccretion disk of gas and dust rotating in a quasi - keplerian manner.

Black hole accretion flows have different regimes, and under various physical conditions, can realize them, demonstrate a great variety of accretion regimes; To provide a detailed description of the core mechanisms that define the nature of accretion flows, many analytical models have been developed; including relatively low accretion rates characterize the Shakura-Sanyaev disk (SSD) and assumes that viscous heating is compensated by local radiative cooling of the disk, a disk that is geometrically thin and optically thick in its outer regions; also known as; standard disc model[1].

ADAF is primarily characterized by its behavior that is contrary to the Standard Model system. Of course, the accretion rate is low, but the disk is characterized by a geometrically thick structure and an optically thin inner part.The accretion flow in this system is dominated by advection [2,3,4,5,6,7,8]. At high accretion rates, the disk exhibits a high optical depth, and the radiation generated by the accretion flow to be trapped within the disk, the photons responsible for carrying the majority of the internal energy are fully trapped within the inflowing material and are unable to escape as radiation. If all these conditions are available during the accretion process, we refer to this model as a slim disc.[9,10,11]. In contrast, at a low mass accretion rate, the disk maintains a geometrically thin structure, and its outer region is optically thin; we can call this type of model a SLE disc.[12].

Most of the models mentioned above do not take into account the effect of disk winds or outflows, while these outflows have been observed a lot in accreting systems composed of compact objects, such as Galactic X-ray binaries [13].The effect of these outflows has also been observed in many accretion systems, which include systems known as cataclysmic variables[14],also in micro-quasars, YSOs [15,16].It has been observed to play a significant role in many other accretion systems[17,18].Winds are also found to be essential in regulating the interaction between active galaxies and their surrounding host galaxies[19,20].Many previous studies have shown that the disk canlose mass, angular momentum, and thermal energy as a consequence of the outflow effect[21,22,23].Many analytical models have studied how hydrodynamic winds affect the structure of ADAFs[24,25,26].To develop solutions that describe the behaviour of outflows in analytical models, we adoptthe assumption thatthe accretion rate varies following a power law function of the radius[27].

There are many works on geometrically thin disks; work has presented a number of global transonic ADAF solution examples, which are characterized by their external connection to geometrically thin discs[28].Another work also constructed the whole family of global ADAF thin disk solutions[29].

Some consider that local radiative cooling resulting from thermal bremsstrahlung[30].On the other hand, some have using bremsstrahlung cooling; the study showed that only an outer (SLE) disk could be smoothly connected to an inner ADAF[31].In these SLE – ADAF solutions, the flow remains optically thin everywhere. Given all of the above, we will conduct an analytical study in this paper that takes into account disc winds/ outflows, and we will also take into account the assumptions in [27].

Our study focuses specifically on regions dominated by radiative cooling by identifying the behaviour of configurations corresponding to thermal balancefor accretion flows around black holesbased on the effect of outflow processesin these regions. Under the influence of these outflows, we will also discuss some of the temperaturebehavior of accretion diskunder both optically thin and thick conditions.

Thispaper is organized as follows: Section 2 describes fundamental equations of the model. Section 3 describes the physical and thermal properties of the disc, Section 4 presents the results obtainedfrom numerical solution, and Section 5 presents the conclusions and discussion.

2. The basic equations

Before examining the behavior of thermal equilibrium and its associated thermal properties, we must describe the fundamental equations of this study. This section focuses on an axisymmetric accretion flow in the context of the pseudo-Newtonian potential [32].

$$\Phi = -GM/(R - R_G)$$

Where G is defined as the gravitational constant, Mis mass of the compact object (black hole), while $R_G = 2GM/c^2$refers to the Schwarzschild radius.

The expression that describes the disc's vertical half – thickness is expressed as$H = c_s/\Omega_k$, note that $c_s = \sqrt{P/\rho}$represents the sound speed under isothermal conditions, at the equatorial plane, Prefers to the sum of overall pressure, and ρcorresponds to the mass density;considering the speed Ω_k, it represents the Keplerian angular velocity determined through the Newtonian potential$\Omega_k = \sqrt{GM/R(R - R_G)^2}$

For the angular velocity, we can find it as follows: $\Omega = \sqrt{GM/R^3}/q$

Thus for keplerian disks, $q = 1 - \frac{R_G}{R}$

The viscosity coefficient, expressed by the following, is the most crucial constant that characterizes the accretion process: $\nu = \frac{2}{3}\alpha c_s H$[1]

Where α is a constant

The parameter of great importance when studying the accretion phenomenon is defined as the surface densityΣ , which corresponds to the density ρ in the following form $\Sigma = 2H\rho$

The total pressure, in this case, is the combined contribution of gas and radiation pressure

$$P = P_{gas} + P_{rad}$$

The expressions for the gas and radiation pressures, respectively, are represented as:[6]

$P_{gas} = \frac{\rho k_B}{\mu m_p}(T_i + T_e)(1)$

and

$$P_{rad} = \frac{Q_-}{4c}\left(\tau + \frac{2}{\sqrt{3}}\right) \quad (2)$$

Where $k_B = 1.38 \times 10^{-16} \mathrm{erg\ k^{-1}}$ is the constant of the Boltzmann; $\mu = 0.617$, it is the constant that expresses the mean molecular weight; the two temperatures of ion and electron, respectively, are T_i and T_e , they are related via $T_e = \min(T_i, 6 \times 10^9 k)$.

Q_- is the rate of radiative cooling that we will discuss below; $\tau = \kappa\rho H$ is the relationship that represents the overall optical depth,however the opacity κ is composed of electron scattering and absorptionopacities$\kappa = \kappa_{es} + \kappa_{abs}$

Here $\kappa_{es} = 0.34\ \mathrm{cm^2 g^{-1}}$ and $\kappa_{abs} = 0.27 \times 10^{25} \rho T_e^{-3.5} \mathrm{cm^2 g^{-1}}$

After we have discussed the most critical parameters in this study, we will discuss some basic equations in the following.

Knowing that the radial velocity V_R , the continuity equation is

$$\frac{1}{R}\frac{d}{dR}(R\Sigma V_R) + \frac{1}{2\pi R}\frac{d\dot{M}_w}{dR} = 0 \quad (3)$$

Where$\dot{M}_w$, is the rate of mass loss that expresses the effect of the outflow/wind and is represented by [22]

$$\dot{M}_w(R) = \int_{R_{in}}^{R} 4\pi R' \dot{m}_w(R') dR' \quad (4)$$

Where R_{in}represents the inner edge radius of the accretion disk, than $\dot{m}_w$represents the rate of mass loss per unit surface area, describing the impact from each side of the disk.

The rate of accretion can be defined as [27]

$$\dot{M} = -2\pi R\Sigma V_R = \dot{M}_0 \left(\frac{R}{R_0}\right)^s \quad (5)$$

Here $\dot{M}_0$, corresponds to the mass accretion rate at a given radiusR_0, but $s$denotes the power-law index, which remains constant parameter.

WhenEq.(4) and Eq.(5)are substituted into Eq.(3), we obtain

$$\dot{m}_w = \frac{\dot{M}s}{4\pi R^2} \quad (6)$$

The constant index of power law $s$depends to the disk thicknessthrough[33]

$$s = \lambda(H/R) \quad (7)$$

Where λ is a constant

After the continuity equation, there are two fundamental equations, the equations corresponding to radial momentum and the azimuthal motion, written as follows:[2]

$$V_R \frac{dV_R}{dR} - \Omega^2 R = -\Omega_k^2 R - \frac{1}{\rho}\frac{d}{dR}(\rho c_s^2) \qquad (8)$$

$$-\frac{1}{R}\frac{d}{dR}(R\Sigma V_R R^2 \Omega) + \frac{1}{R}\frac{d}{dR}\left(R^3 \nu \Sigma \frac{d\Omega}{dR}\right) - \frac{(lR)^2\Omega}{2\pi R}\frac{d\dot{M}_w}{dR} = 0 \qquad (9)$$

The term appearing laston the left in the azimuthal motion equationindicates the angular momentum transported by the outflowing material.

When $l > 1$.This is associated to magnetic disk winds generated by centrifugal effects, which contribute to a more efficient angular momentum loss from the disk.

In this study, it is always assumed that the accretion disk structures are self-similar.

We substitute Eq. (3)into Eq. (9), integrate from $3R_G$ to$R$, and get

$$\nu\Sigma = \frac{\dot{M} f g^{-1}}{3\pi}\left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right) \qquad (10)$$

Where $g = -\frac{2}{3}\frac{d\ln\Omega_k}{d\ln R}$ and $f = 1 - [\Omega(3R_G)/\Omega(R)](3R_G/R)^{s+2}$

3. Physical and thermal properties of the disk

The energy conservation equation is represented through the equilibrium betweenthe rate of viscous heatingQ_+ , radiative cooling rateQ_- , and the overall heat transfer via radial advectionQ_{adv}

It is expressed as

$$Q_+ = Q_- + Q_{adv} \qquad (11)$$

We may represent the rates of heating and cooling terms in the energy equation as follows:[2,5,34]

The viscous heating rate is given by the following expression:

$$Q_+ = \nu\Sigma\left(R\frac{d\Omega}{dR}\right)^2 \qquad (12)$$

The expression describing the rate of cooling through advection can be written as follows:

$$Q_{adv} = \frac{\dot{M}}{2\pi R^2}\frac{P}{\rho}\xi \qquad (13)$$

Where

$$\xi = \left[\frac{4-3\beta}{\Gamma_3 - 1}\frac{d\ln T}{d\ln R} - (4-3\beta)\frac{d\ln\Sigma}{d\ln R}\right] \qquad (14)$$

Here $\beta = P_g/P$is a constant quantity that represents the ratio between gas pressure and the total pressure, $\Gamma_3 = 1 + (4 - 3\beta)(\gamma - 1)/[\beta + 12(\gamma - 1)(\beta - 1)]$, and $\gamma$is defined as the ratio of specific heats. We take $\beta = 1$ and $\Gamma_3 = \gamma$

The radiative cooling rate can be formulated as follows:

$$Q_- = \frac{4\sigma T_e^4}{\frac{3\tau}{2}+\sqrt{3}+\frac{1}{\tau_{abs}}} \quad (15)$$

Here,τindicates the total optical depth along the vertical direction, measured from the disc midplane to its surface,$\tau_{abs}$is the optical depth, but the part that expresses absorption. $T_e$is defined as the electron temperature located at the equatorial plane. Here $\tau = \tau_{abs} + \tau_{es}$

$\tau_{es}$is the part that expresses scattering, for the optical depth.

In case, the gas is strongly optically thick, the radiative coolingrate can be given by this formula:[4]

$$Q_- = \frac{8}{3}\frac{\sigma T_e^4}{\tau} \quad (16)$$

When we use the Rosseland, it means opacity κ_R , the optical depth $\tau = \kappa_R \rho H = \kappa_R \Sigma$

For an optically thin gas, the bremsstrahlung radiationcan be represented as follows:

$$Q_- = Q_{brem} = 1.24 \times 10^{21} H \rho^2 T^{1/2} \text{ ergs s}^{-1}\text{cm}^{-2} \quad (17)$$

We need some scaled numerical relations to describe the self-similar solutions to the above solutions

$M = mM_\odot$, $R = rR_G$,$q = 1 - \frac{R_G}{R}$, $\dot{m} = \dot{M}/\dot{M}_{Edd}$, where $\dot{M}_{Edd} = \frac{L_{Edd}}{c^2} = \frac{4\pi cGM}{c^2\kappa_{es}}$is the rate of the mass accretion of the Eddington. (κ_{es} Correspondsthe opacity produced by electron scattering). Where $M_\odot$denotes the solar mass

Following some calculations, we find that the term advection cooling takes the following form:

$$Q_{adv} = \frac{\dot{M}}{2\pi R^2} c_s^2 \xi \quad (18)$$

$$Q_{adv} = \frac{c\kappa_{es}}{2R}\frac{1}{q^2}\left(\frac{cR_G}{\kappa_{es}R}\right)^2 \dot{m}\left(\frac{H}{R}\right)^2 \xi \qquad (19)$$

We can also reduce the equation of viscous heating rateto the following algebraic form:

$$Q_+ = \frac{3\dot{M}\Omega^2 fg^{-1}}{4\pi}\left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right) \qquad (20)$$

$$Q_+ = \frac{3}{2}\frac{c\kappa_{es}}{2R}\frac{1}{q^2}\left(\frac{cR_G}{\kappa_{es}R}\right)^2 \dot{m}\, fg^{-1}\left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right) \qquad (21)$$

We assume $\kappa_R = \kappa_{es}$,the radiative cooling rate for the optically thick casetakes the form:

$$Q_- = \frac{4c}{\kappa_{es}}\Omega_k^{3/2}\left(\frac{R_G}{2R}\right)^{1/4}\sqrt{Rc}.\frac{1}{\sqrt{q}}\left(\frac{H}{R}\right) \qquad (22)$$

Under the same assumption as above, the bremsstrahlung radiation for the optical thin case can be written as follows:

$$Q_- = 3.4 \times 10^{-6}\left(\frac{cR_G}{\kappa_{es}R}\right)^2\left(\frac{R}{R_G}\right)^2 \Omega_k \alpha^{-2}(\alpha\Sigma)^2 \qquad (23)$$

We have used the equationEq.(10) to get the standard equation:

$$\left(\frac{H}{R}\right)^2 = \frac{\sqrt{2}}{\kappa_{es}} q\left(\frac{R_G}{R}\right)^{1/2} \dot{m}(\alpha\Sigma)^{-1} fg^{-1}\left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right) (24)$$

When we apply the standard equation; the advection cooling rate term can be written as

$$Q_{adv} = \left(\frac{cR_G}{\kappa_{es}R}\right)^2 \Omega_k \xi \dot{m}^2 (\alpha\Sigma)^{-1} fg^{-1}\left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right) \qquad (25)$$

The equation of the radiative cooling rate for the optically thick can be written as

$$Q_- = 4\left(\frac{cR_G}{\kappa_{es}R}\right)^2 \Omega_k^{3/2}\left(\frac{\kappa_{es}R_G}{c}\right)^{1/2}\left(\frac{R}{R_G}\right)^2 (\dot{m})^{1/2}(\alpha\Sigma)^{-1/2}\left[fg^{-1}\left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right)\right]^{1/2} \qquad (26)$$

In the above equations, we saw the most critical terms that characterize energy conservation; we will study inlast part of this section the temperature properties of the disc.

To examine the effect of the outflow parameters on the temperature behavior of the disk applicable to both optically thin and thick accretion disks,our study designed to achieve the following.

Can be obtained the radial temperature properties of optically thin accretion discs from Eq. (10), and we will consider two solutions, in the first one, advection cooling dominates completely and the other when local radiative cooling dominates

We can writEq.(10) as the following

$$\frac{2}{3}\alpha c_s H.\Sigma = \frac{\dot{M} f g^{-1}}{3\pi}\left(1-\frac{l^2 s}{s+\frac{1}{2}}\right) \quad (27)$$

and

$$\frac{2}{3}c_s^2(\alpha\Sigma) = \Omega_k \frac{1}{3\pi}\dot{m}.\frac{4\pi GM}{c\kappa_{es}} f g^{-1}\left(1-\frac{l^2 s}{s+\frac{1}{2}}\right) \quad (28)$$

We can find that

$$T = 1.39\times 10^{13}\,\dot{m}(\alpha\Sigma)^{-1}\left(\frac{R}{R_G}\right)^{-3/2} f g^{-1}\left(1-\frac{l^2 s}{s+\frac{1}{2}}\right) \quad (29)$$

In the first solution, where advection cooling dominates, the temperature does not depend on the rate of the mass accretion

$$T = 0.502\times 10^{13}\left(\frac{R}{R_G}\right)^{-1} q^{-2} f g^{-1}\left(1-\frac{l^2 s}{s+\frac{1}{2}}\right) \quad (30)$$

In the second solution, where local radiative cooling dominates, the temperature depends on the viscosity coefficient and the rate of the mass accretion.

$$T = 4.295\times 10^{10}\dot{m}^{1/2}\alpha^{-1}\left(\frac{R}{R_G}\right)^{-3/4} q^{-1/2} f^{1/2} g^{-1/2}\left(1-\frac{l^2 s}{s+\frac{1}{2}}\right)^{1/2} \quad (31)$$

To study the behaviour of the disk's temperature in the optically thick accretion disk, we can use Eq.(16).It can be written as[35,36]

$$T = 10^5\left(\frac{M_\odot}{M}\right)^{1/4}\dot{m}^{1/4}\left(\frac{R}{R_G}\right)^{-3/4} f^{1/4} g^{-1/4}\left(1-\frac{l^2 s}{s+\frac{1}{2}}\right)^{1/4} \quad (32)$$

4. Numerical results

4.1.The thermal equilibrium solutions

The accretion disk and the study of its thermal equilibrium solutions behaviour are fundamental properties in this work; it is convenient to discuss the results in the $\log(\dot{M}/\dot{M}_{Edd})$ vs. $\log\Sigma$ Plane;the accretion flow of a black hole with its surrounding accretion disk have different branches; we obtain a different curves show the solutions of these branches.

Our equations were solved for various choices of the different parameters: the coefficient of viscosity α , the outflows parameters s and l , and of course, we must not forget the radius R.

We plot the $\log(\dot{M}/\dot{M}_{Edd})$ vs. $\log\Sigma$, for different outflow parameters s , and l , the curves show the role of the outflow at the different viscosity coefficients α

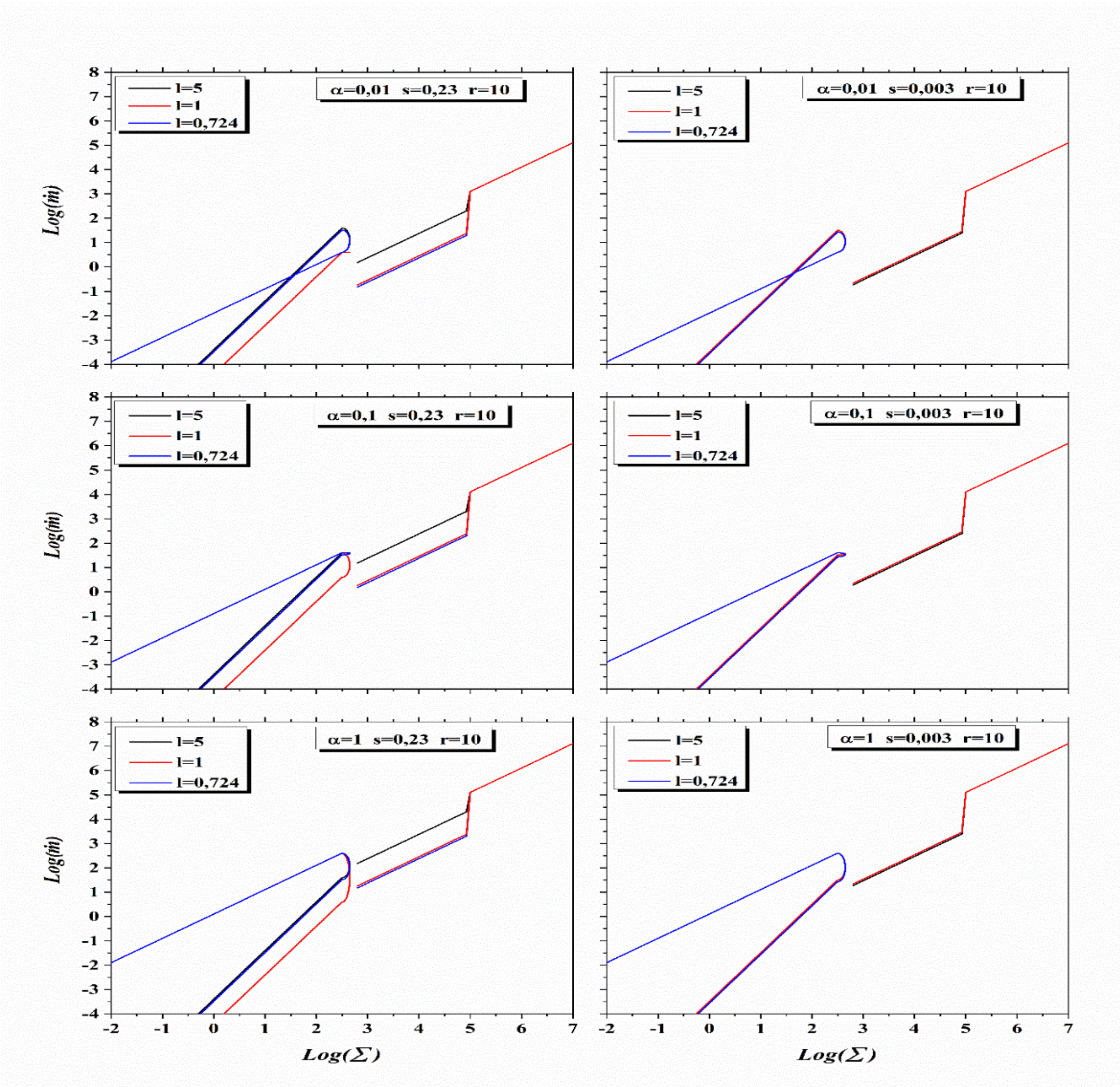

Fig.1. Thermal equilibria of accretion disks, with different values of s and$l$, it's plottedfor $\alpha = 0.01,\ 0.1$ and$\alpha = 1$, respectively. Red, blue, and black lines explain the results for ($l = 1$), ($l = 0.742$), and ($l = 5$), respectively.

All solutions of the Fig.1, at given radius $R = 10R_G$, than mass of black holeis$M = 10M_\odot$

Fig. 1 shows the thermal equilibrium solutions of the accretion disk. In the figure, curves on the left have two branches; upper branch represents advection-dominated solution, but lowerbranch represents SLE disk. On the other handinthe right, all S-shaped curves, upper part for the slim disks, and the lower part for the standard disc model SSD.

Separate

We can see from Figure for$\alpha = 0.1$, SSD branch appears and separates from the ADAF branch at average accretion rates ($1 < \dot{m} < 10^2$), but from Figure of $\alpha = 1$ it is found that the two branches emerge and decouple at moderate accretion rates ($10^2 < \dot{m} < 10^3$), but of $\alpha = 0.01$ The SSD branch exceed ADAF branch at $\dot{m}{\sim}1$

In the curves on the right, the move from SSD until slim disk at ($\dot{m}{\sim}10^4$) from the figures of ($\alpha = 0.1$), and ($\dot{m} < 10^4$) for the figures of ($\alpha = 0.01$), However at $\dot{m} > 10^4$ in the figures of ($\alpha = 1$)

For the SSD branch, the curves of the values ($s = 0.003$) are in good consistently, but of ($s = 0.23$) there is a difference when increasing the effect of outflows $l = 5$

We can decide that therate of mass accretion increaseswhen the viscosity coefficient α increases. At a higher value of the viscosity coefficient ($\alpha = 1$), the two branches of the outer regions are getting closer; we also see that the effects of outflow are visible in the behavior of the outer areas.

In the next figure, we will plot the variations of mass accretion rate with surface density for different outflow parameters s , and l ,which representthe role of the outflow at the different radii r

In all solutions of this figure, viscosity coefficient at$\alpha = 0.1$, than mass of black holeis $M = 10M_\odot$

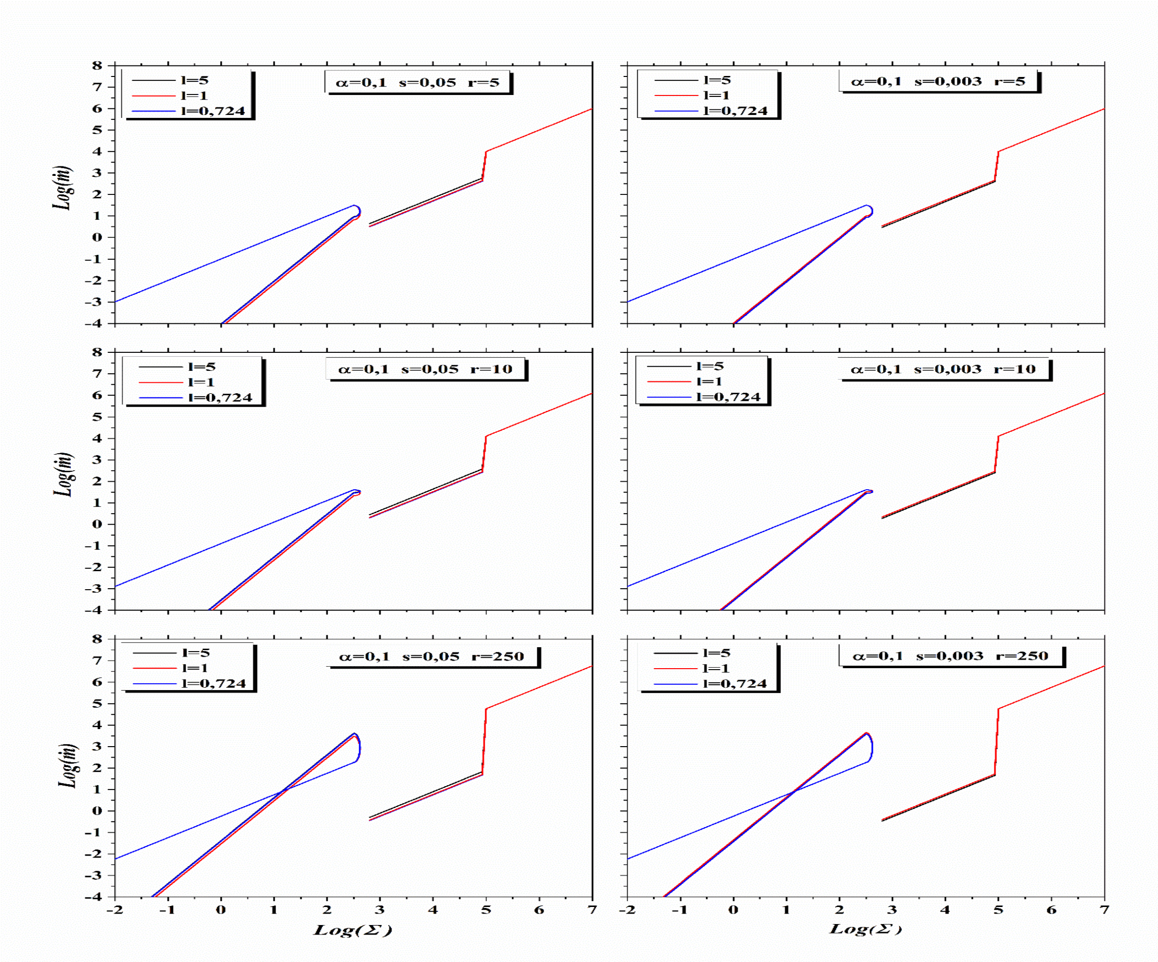

Fig.2. Thermal equilibria of accretion disks of $\alpha = 0.1$, with different values of s and l are plotted at $R/R_G = 5, 10$ and $R/R_G = 250$,respectively. Red, blue, and black lines illustrate the results for (l = 1), (l = 0.742), and (l = 5), respectively.

Fig.2 illustrates the behavior of changes in the rate of mass accretion as a function of surface density for two values of parameter s as well three values of parameters l ,representing the role of the outflow at different radii. All the branches we saw previously appear in the curves at different radius, showing the previous solutions for each ofADAF, SLE, SSD and slim disk.

In these solutions at $R = 5R_G$ and $R = 10R_G$,SSD branch appears and couples with ADAF branch at average accretion rates($10 < \dot{m} < 10^2$), but at $R = 250R_G$, SSD branch exceeds ADAF branch in $\dot{m} \sim 10$

The curves' behaviour shows that slim disk solutions correspond to rise accretion rates, while ADAF/SSD solutions appear in least accretion rates. However, SSD solutions are convenient at large radii, while ADAF solutions are suitable for small radii.

At large radii, $R = 250R_G$, there is a long trend toward transitioning from SSDs to slim disks; due tofarther away from black hole, we see that outflows have a significant and substantial impact ondisk structures, especially (outer region) .

Our model solutions demonstrate role of outflows in general structure of the disk, with main difference in SSD , SLE branches, which shows increased outflows in these regions, something not seen in other models.

This study shows that behavior of thermal equilibrium solutions of accretion disk in the radiative cooling regions at relatively small values ($s = 0.05$) and ($s = 0.003$) is optimal, and the outflows are essential effects in the radiative cooling dominated areas which represent the SSD and SLE solutions.

4.2. Disk temperature properties

In this content, we will study some important temperature characteristics as a function of the accretion radius in both optical (thin and thick) cases of accretion disk flows.

We know that in the first case, when the accretion disk flows optically thin, the disk temperature depends on rate of mass accretion, where local radiative cooling dominates. To study the behavior of this case, we can use both EquationsEq. (30)than Eq. (31).

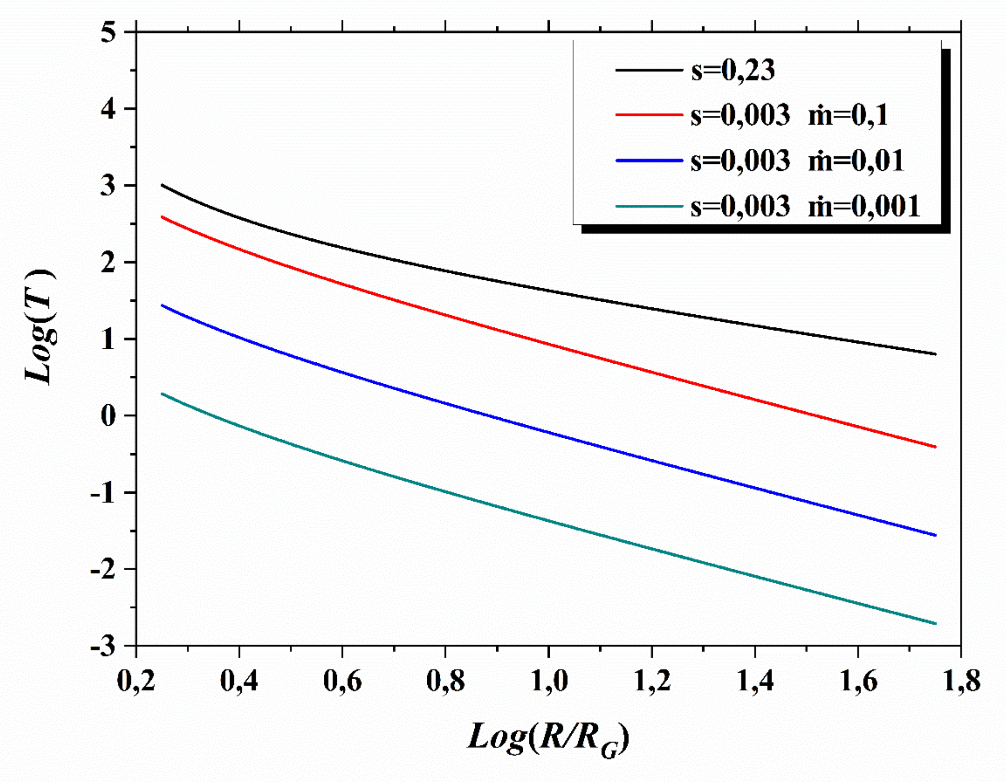

Fig.3. The radial temperature of optically thin accretion disks for $\alpha = 0.1$, with different values of s . Black line represents advection dominated solutions $s = 0.23$. Red, blue and green lines representresults for $\dot{m} = 0.1,\ 0.01$ and$\dot{m} = 0.001$ respectively of local radiative cooling dominated solutions $s = 0.003$.

Fig.3 shows that the temperature where advection dominates (ADAF) is higher than in the case that describes the standard model (SSD). We can also see that in regions where radiative cooling prevails,the curves indicate that the decrease in disk temperature corresponds to a decrease in rate of mass accretion values .

We find that radial temperature shape concerning SSD solutions in the regions where radiative cooling is dominant is in good agreement with $a\left(\frac{R}{R_G}\right)^{-3/4}$ relation for all cases where disc is radiatively thin[1].

In the second case, when the accretion disk flows optically thick, the disk temperature depends on rate of mass accretionthan mass of black hole equal$M = 10M_{\odot}$. To study the behavior of this case, we can use Eq. (32).

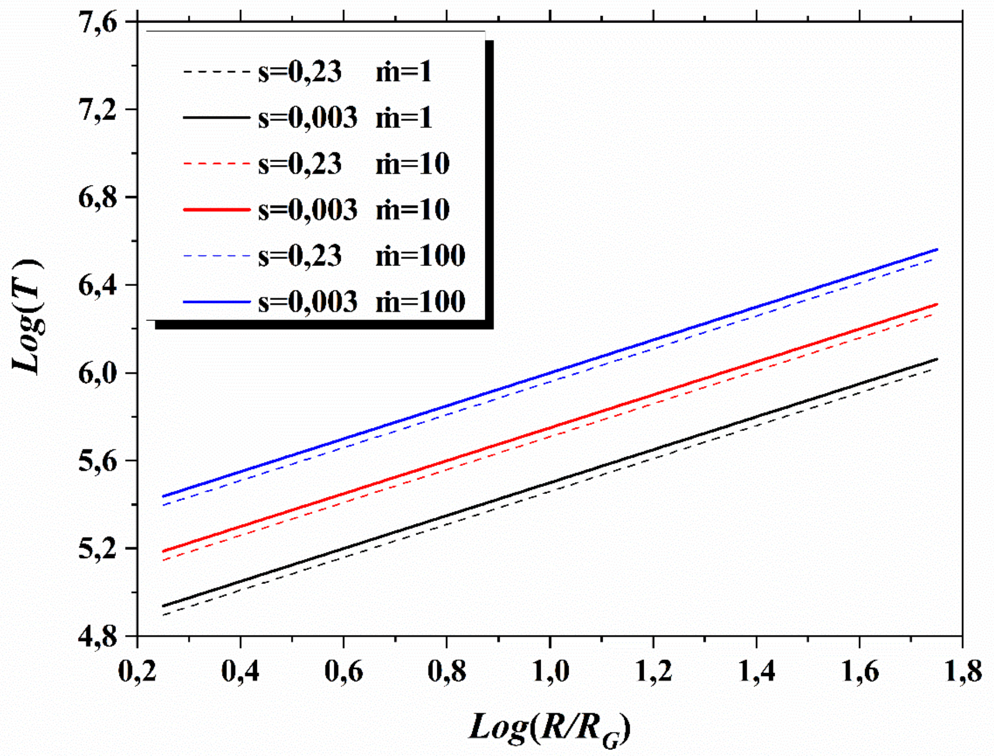

Fig.4. Structure of temperature of optically thick accretion disk for $\alpha = 0.1$, with different values of s . Black, red and blue lines represent results for $\dot{m} = 1,\ 10$ and $\dot{m} = 100$ respectively. The dotted lines for $s = 0.23$ and the solid lines for$s = 0.003$.

Fig.4shows temperature changes as a function of the accretion radius.Temperature profiles of the optically thick solution in agreementa$\left(\frac{R}{R_G}\right)^{-3/4}$ relation.The temperature for $s = 0.003$ solutions is higher than for $s = 0.23$solutions.When the rate of mass accretion increases, the temperature of the optically thick accretion disk grows linearly.Temperature properties are similar to those familiar from Sakura- Sunyaev models.

We have examined the outflows role in the accretion disk properties in both its optically cases.

In first case, optically thin accretion disk, temperature properties depend on rate of mass accretion, butaccretion disk in optically thick case depends on rate of mass accretion and mass of black hole.

This study revealed that the temperature in accretion disk flows in both optical (thin, thick) states increases as rate of mass accretion increases.

5. Conclusions and Discussion

In this paper, we have studied an analytical model that takes into account disc winds and outflows, according to the assumptions given in [27].The most important aspect of studying the accretion flow around black holes is identifying some of the behavior of their thermal equilibrium solutions. In our work, we relied on adding the effect of outflows, especially in regions where radiative cooling dominated.We see a new topology type of thermal equilibrium exists for the values different from the viscosity. The results obtained from different values of s and l , showed the important role of outflows on accretion systems. We should remember that extracting more angular momentum from disk is due to these outflows as well, and that more energy affects the disk's structure.

It is clearly shown that the mass accretion rates of flow with the outflow in the outer regions, which are governed by radiative cooling, are significantly higher, and these regions are dominated by SSD and SLE solutions.At large radii, there is a long transition from SSD to slim disk, and due to the greater distance from black hole, the effect of outflows is significant on the disk structure, especially (outer region)

With this study, we were able to support the results of[2, 5], and it was indeed confirmed that ADAF and SSD solutions are sufficient to describe inner and outer regionsrespectively of accretion flow aroundblack hole.Main difference between our model solutions and those of recent works lies in the configuration of the disk structure, especially at the SSD and SLE branches, which indicates improved outflows in these regions.

We studied the temperature distribution in both optically accretion disks under influence of outflow parameters; we found that rate of mass accretion $\dot{m}$has greater importance on thermal properties of accretion discs flows inregions of dominant radiative cooling.We confirm finally that role of the outflow is essential in the behavior of disk accretion flow, especially in regions where flow is dominant (in SSD and SLE solutions) and governed by radiative cooling.

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