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Energy Equations and Thermodynamics of Accretion Flows Around compact Objects

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Abstract

When accretion disks form around compact objects (black hole), part of the energy produced is converted into the disk's kinetic energy, while the remainder is converted into thermal energy. The main objective of this research is to study the behavior of matter transport, particularly convection and radiation, of various accretion flows surrounding a compact object (black hole). However, the second aim of this work is to follow the effect of the mass accretion rate on thermal equilibrium and temperature behaviors of accreting matter around a compact object (black hole), considering both optical states (thin, thick) characterize accretion discs. We will see that the accretion rate and optical depth are the two key determining factors of the observed "properties" of the black hole.

Keywords: Compact object, Thermal energy, Convection, Thermal properties, Radiative cooling.

1. Introduction

In this work, we will study both the thermodynamic equations of accretion flows around a compact body (a black hole) and the temperature behavior of the accretion material. We will also see the fundamental physical difference between accretion fluxes that lose energy through radiation and those that retain it through convection in both thick and thin optical states, and what this reflects about the behavior of matter around a black hole. But before we begin with this, we will mention some works in which some have discussed the phenomenon of accretion.

There are many objects in the universe that are sources of enormous amounts of energy; some binary stars that emit X-rays and certain galactic nuclei are of this type. One of most powerful mechanism in universe that produces huge energy is phenomenon of massive accretion of matter around an object compact (black holes). Studying behavior of this accretion process is fundamental to understanding how galaxies and their black holes evolve together. To understand the mechanism governing to the growth of these structures, we adopt a physical model composed of an accretion disk formed from dust and gas rotating in a quasi - keplerian manner surrounding an object compact (black holes).

Black hole accretion flows have different regimes, and under various physical conditions, can realize them, demonstrate a great variety of accretion regimes; To provide a detailed description of the core mechanisms that define the nature of accretion flows, many analytical models have been developed; in the case of relatively low accretion rates that characterize Shakura-Sannaev disk (SSD), it is assumed that local radiative cooling compensates for disk's viscous heating. Outer regions of these disks are characterized as being geometrically thin as well optically thick; also known as; standard disc model.

A study of the thick case developed the Standard Thin Disk, a geometrically thin and optically thick material, and demonstrated that when a material is sufficiently dense, it reaches thermal equilibrium and radiates like a blackbody [1].

A study of the optically thin case proposed a model of advection-dominated accretion flows and demonstrated that at low accumulation rates, the material becomes optically thin and fails to cool, leading to increased temperature [2, 3].

ADAF is primarily characterized by its behavior that is contrary to the standard disc model, of course, accretion rate also is low, but this disk has a geometrically thick structure and its inner part is optically thin. In this system accretion flow is dominated by advection [4, 5]. At high accretion rate, the disk exhibits a high optical depth, while, the radiation produced by flow of matter is trapped inside the disk, however the photons supervise for carrying the most of internal energy are fully trapped within the inflowing material and are unable to escape as radiation. If all these conditions are available during the accretion process, we refer to this model as a slim disc.

This study links the two cases at very high accretion rates, the slim disk model that explains how heat transfer (advection) becomes important even in optically thick disks when the accretion rate exceeds the Eddington limit. [6]. In contrast, at a low mass accretion rate, the disk maintains a geometrically thin structure, and its outer region is optically thin; we can call this type of model a SLE disc. [7].

Most of the models mentioned above do not take into account the effect of disk winds or outflows, while these outflows have been observed a lot in accreting systems composed of compact objects, such as Galactic X-ray binaries [8]. The effect of these outflows has also been observed in many accretion systems, which include systems known as cataclysmic variables [9], also in micro-quasars, YSOs [10, 11]. It has been observed to play a significant role in many other accretion systems [12, 13]. Winds are also found to be essential in regulating the interaction between active galaxies and their surrounding host galaxies [14, 15]. Many previous studies have shown that the disk can lose both angular momentum and mass; not to remember the loss of heat energy as a consequence of outflow effect

[16,17]. Many analytical models have studied how hydrodynamic winds affect the structure of ADAFs [18, 19].To develop solutions that describe the behaviour of outflows in analytical models, we take into account that rate of accretion changes according to a force law function of the radius [20].

There are many works on geometrically thin disks; work has presented a number of global transonic ADAF solution examples, which are characterized by their external connection to geometrically thin discs [21]. Another work also constructed the whole family of global ADAF thin disk solutions [22]. Some consider that local radiative cooling resulting from thermal bremsstrahlung [23]. On the other hand, some have using bremsstrahlung cooling; the study showed that only an outer (SLE) disk could be smoothly connected to an inner ADAF [24]. In these SLE – ADAF solutions, the flow remains optically thin everywhere. Given all of the above, we will take into account the assumptions in [20]. This study offers not only theoretical contributions, but also elucidates the behavior of configurations corresponding to thermal balance for accretion flows around an object compact (black hole), as well as some thermal behavior of the accretion disk under both optical conditions.

2. The basic equations of the model

Before examining the thermal equilibrium behavior of the accretion disks and its associated thermal properties, we must describe the fundamental equations of this study. This section focuses on an axisymmetric accretion flow in the context of a quasi-Newtonian potential [25].

$$\Phi = -GM/(R - R_G)$$

Here G represents gravitational constant, M denotes mass of compact object (black hole), while $R_G = 2GM/c^2$ refers to Schwarzschild radius.

There is expression that describes the disc's vertical half – thickness as $H = c_s/\Omega_k$, note that $c_s = \sqrt{P/\rho}$ represents the sound speed under isothermal conditions, along equatorial plane, P denotes total pressure, and ρ corresponds to the mass density; considering the speed Ω_k, it denotes Keplerian angular velocity determined through quasi-Newtonian potential $\Omega_k = \sqrt{GM/R(R - R_G)^2}$

For the angular velocity, we can find it as follows: $\Omega = \sqrt{GM/R^3}/q$

Thus for keplerian disks, $q = 1 - \frac{R_G}{R}$

The viscosity coefficient, expressed by the following, is the most crucial constant that characterizes the accretion process: $\nu = \frac{2}{3}\alpha c_s H$ [1]

Here α is fixed value

The specified parameter of great importance when studying the accretion phenomenon is defined as the surface density Σ , which corresponds to the density ρ in the following form $\Sigma = 2H\rho$

The overall pressure, during this case, it results from combined contribution of gas pressure plus radiation pressure $P = P_{gas} + P_{rad}$

The expressions for the gas and radiation pressures, respectively, are represented as: [5]

$$P_{gas} = \frac{\rho k_B}{\mu m_p}(T_i + T_e)$$

(1)

and

$$P_{rad} = \frac{Q_-}{4c}\left(\tau + \frac{2}{\sqrt{3}}\right)$$

(2)

Where $k_B = 1.38 \times 10^{-16} \text{erg k}^{-1}$ denotes Boltzmann constant, $\mu = 0.617$ it is the constant that expresses molecular weight average, two temperatures from electrons and ions, respectively, are T_e and T_i , they are related via $T_e = \min(T_i, 6 \times 10^9 \text{k})$.

Q_- is the rate of radiative cooling that we will discuss below; $\tau = \kappa\rho H$ is the relationship that represents the overall optical depth, however the opacity κ is composed of electron scattering and absorption opacities $\kappa = \kappa_{es} + \kappa_{abs}$

Here $\kappa_{es} = 0.34 \text{ cm}^2\text{g}^{-1}$ and $\kappa_{abs} = 0.27 \times 10^{25}\rho T_e^{-3.5}\text{cm}^2\text{g}^{-1}$

After we have discussed the most critical parameters in this study, we will discuss some basic equations in the following.

Knowing that the radial velocity V_R , the continuity equation is

$$\frac{1}{R}\frac{d}{dR}(R\Sigma V_R) + \frac{1}{2\pi R}\frac{d\dot{M}_w}{dR} = 0$$

(3)

Where $\dot{M}_w$, represents loss rate of mass, that expresses effect of outflow/wind, is described by the following relationship. [17]

$$\dot{M}_w(R) = \int_{R_{in}}^{R} 4\pi R' \dot{m}_w(R')dR'$$

(4)

Where R_{in} corresponds to disk's inner-edge radius, whereas $\dot{m}_w$ represents loss rate of mass per unit surface area, describing impact from each side of the disk.

The rate of accretion can be defined as [20]

$$\dot{M} = -2\pi R \Sigma V_R = \dot{M}_0 \left(\frac{R}{R_0}\right)^s$$

(5)

Here $\dot{M}_0$, corresponds to rate of mass accretion at a given radius R_0, but s denotes power-law index, which remains constant parameter.

When Eq. (4) and Eq. (5) are substituted into Eq. (3), we obtain

$$\dot{m}_w = \frac{\dot{M}s}{4\pi R^2}$$

(6)

The constant index of power law s depends to the disk thickness through [26]

$$s = \lambda(H/R)$$

(7)

Here λ has a constant value

After the continuity equation, there are two fundamental equations, the equations corresponding to radial momentum and the azimuthal motion, written as follows: [2]

$$V_R \frac{dV_R}{dR} - \Omega^2 R = -\Omega_k^2 R - \frac{1}{\rho}\frac{d}{dR}(\rho c_s^2)$$

(8)

$$-\frac{1}{R}\frac{d}{dR}(R\Sigma V_R R^2 \Omega) + \frac{1}{R}\frac{d}{dR}\left(R^3 \nu \Sigma \frac{d\Omega}{dR}\right) - \frac{(lR)^2\Omega}{2\pi R}\frac{d\dot{M}_w}{dR} = 0$$

(9)

The term appearing last on the left in the azimuthal motion equation indicates the angular momentum transported by the outflowing material.

When $l > 1$. This is associated to magnetic disk winds generated by centrifugal effects; this contributes most to loss of angular momentum from disk.

During this study, it is always assumed that structures of accretion disk are self-similar.

We substitute Eq. (3) into Eq. (9), integrate from $3R_G$ to R, and get

$$\nu\Sigma = \frac{\dot{M}fg^{-1}}{3\pi}\left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right)$$

(10)

Where $g = -\frac{2}{3}\frac{d\ln\Omega_k}{d\ln R}$ and $f = 1 - [\Omega(3R_G)/\Omega(R)](3R_G/R)^{s+2}$

3. Thermal properties of the accretion disk

The equation that describes energy conservation is represented by balancing between rate of viscous heating Q_+ , radiative cooling rate Q_- , and total heat transfer via radial advection Q_{adv}

It is expressed as

$$Q_+ = Q_- + Q_{adv}$$

(11)

We may represent the rates of heating and cooling terms in the energy equation as follows: [2, 4]

The viscous heating rate is given by the following expression:

$$Q_+ = \nu\Sigma\left(R\frac{d\Omega}{dR}\right)^2$$

(12)

The expression describing the rate of cooling through advection can be written as follows:

$$Q_{adv} = \frac{\dot{M}}{2\pi R^2}\frac{P}{\rho}\xi$$

(13)

Where

$$\xi = \left[\frac{4-3\beta}{\Gamma_3-1}\frac{d\ln T}{d\ln R} - (4-3\beta)\frac{d\ln\Sigma}{d\ln R}\right]$$

(14)

Here $\beta = P_g/P$ is a constant quantity that represents ratio between both gas pressure and total pressure,

, but γ represents ratio of specific heats$\Gamma_3 = 1 + (4-3\beta)(\gamma-1)/[\beta + 12(\gamma-1)(\beta-1)]$

We take $\beta = 1$ and $\Gamma_3 = \gamma$

The radiative cooling rate can be formulated as follows:

$$Q_- = \frac{4\sigma T_e^4}{\frac{3\tau}{2}+\sqrt{3}+\frac{1}{\tau_{abs}}}$$

(15)

Here, τ indicates the total optical depth along the vertical direction, this parameter is measured from middle level of the disc to its surface, τ_{abs} represents optical depth, specifically the component that expresses absorption. T_e is defined as the electron temperature located at the equatorial plane. Here

$\tau = \tau_{abs} + \tau_{es}$

it also relates to optical depth, but the part that expresses dispersionτ_{es}

When an optically thick gas, rate of radiative cooling can be given as follows: [3]

$$Q_- = \frac{8}{3}\frac{\sigma T_e^4}{\tau}$$

(16)

When we use the Rosseland, we can write $\tau = \kappa_R \rho H = \kappa_R \Sigma$, here κ_R it means opacity

For an optically thin gas, the bremsstrahlung radiation can be represented as follows:

$$Q_- = Q_{brem} = 1.24 \times 10^{21} H \rho^2 T^{1/2} \text{ ergs s}^{-1} \text{cm}^{-2}$$

(17)

We need some scaled numerical relations to describe self-similar solutions to above solutions

$M = mM_\odot$, $R = rR_G$, $q = 1 - \frac{R_G}{R}$, $\dot{m} = \dot{M}/\dot{M}_{Edd}$, where $\dot{M}_{Edd} = \frac{L_{Edd}}{c^2} = \frac{4\pi c G M}{c^2 \kappa_{es}}$ is rate of mass accretion of Eddington . (κ_{es} Corresponds the opacity produced by electron scattering).

Where $M_\odot$ denotes the solar mass.

Following some calculations, we find that the term advection cooling takes the following form:

$$Q_{adv} = \frac{\dot{M}}{2\pi R^2} c_s^2 \xi$$

(18)

$$Q_{adv} = \frac{c\kappa_{es}}{2R}\frac{1}{q^2}\left(\frac{cR_G}{\kappa_{es}R}\right)^2 \dot{m}\left(\frac{H}{R}\right)^2 \xi$$

(19)

We can also reduce the equation of viscous heating rate to the following algebraic form:

$$Q_+ = \frac{3\dot{M}\Omega^2 f g^{-1}}{4\pi}\left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right)$$

(20)

$$Q_+ = \frac{3}{2}\frac{c\kappa_{es}}{2R}\frac{1}{q^2}\left(\frac{cR_G}{\kappa_{es}R}\right)^2 \dot{m}\, f g^{-1}\left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right)$$

(21)

We assume $\kappa_R = \kappa_{es}$, the radiative cooling rate for the optically thick case takes the form:

$$Q_- = \frac{4c}{\kappa_{es}} \Omega_k^{3/2} \left(\frac{R_G}{2R}\right)^{1/4} \sqrt{Rc}.\frac{1}{\sqrt{q}}\left(\frac{H}{R}\right)$$

(22)

Under the same assumption as above, the bremsstrahlung radiation for the optical thin case can be written as follows:

$$Q_- = 3.4 \times 10^{-6} \left(\frac{cR_G}{\kappa_{es}R}\right)^2 \left(\frac{R}{R_G}\right)^2 \Omega_k \alpha^{-2} (\alpha\Sigma)^2$$

(23)

We have used the equation Eq. (10) to get the standard equation:

$$\left(\frac{H}{R}\right)^2 = \frac{\sqrt{2}}{\kappa_{es}} q \left(\frac{R_G}{R}\right)^{1/2} \dot{m} (\alpha\Sigma)^{-1} fg^{-1} \left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right)$$

(24)

When we apply the standard equation; the advection cooling rate term can be written as

$$Q_{adv} = \left(\frac{cR_G}{\kappa_{es}R}\right)^2 \Omega_k \xi \dot{m}^2 (\alpha\Sigma)^{-1} fg^{-1} \left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right)$$

(25)

The equation of the radiative cooling rate for the optically thick can be written as

$$Q_- = 4 \left(\frac{cR_G}{\kappa_{es}R}\right)^2 \Omega_k^{3/2} \left(\frac{\kappa_{es}R_G}{c}\right)^{1/2} \left(\frac{R}{R_G}\right)^2 (\dot{m})^{1/2} (\alpha\Sigma)^{-1/2} \left[fg^{-1}\left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right)\right]^{1/2}$$

(26)

In the above equations, we saw the most critical terms that characterize energy conservation;

We will study in last part of this section the temperature properties of the disc.

To examine effect of outflow parameters on temperature behavior of disk applicable to both optically thin and thick accretion disks, our study designed to achieve the following.

We can obtain the radial temperature properties of optically thin accretion discs from Eq. (10), and we will consider two solutions, in the first one, advection cooling dominates completely and the other when local radiative cooling dominates

We can write Eq. (10) as the following

$$\frac{2}{3} \alpha c_s H.\Sigma = \frac{\dot{M} fg^{-1}}{3\pi} \left(1 - \frac{l^2 s}{s+\frac{1}{2}}\right)$$

(27)

and

$$\frac{2}{3}c_s^2(\alpha\Sigma) = \Omega_k \frac{1}{3\pi}\dot{m}.\frac{4\pi GM}{c\kappa_{es}} fg^{-1}\left(1-\frac{l^2 s}{s+\frac{1}{2}}\right)$$

(28)

We can find that

$$T = 1.39\times 10^{13}\ \dot{m}(\alpha\Sigma)^{-1}\left(\frac{R}{R_G}\right)^{-3/2} fg^{-1}\left(1-\frac{l^2 s}{s+\frac{1}{2}}\right)$$

(29)

In the first solution, where advection cooling dominates, the temperature does not depend on the rate of the mass accretion

$$T = 0.502\times 10^{13}\left(\frac{R}{R_G}\right)^{-1} q^{-2}fg^{-1}\left(1-\frac{l^2 s}{s+\frac{1}{2}}\right)$$

(30)

In the second solution, where local radiative cooling dominates, the temperature depends on the viscosity coefficient and the rate of the mass accretion.

$$T = 4.295\times 10^{10}\dot{m}^{1/2}\alpha^{-1}\left(\frac{R}{R_G}\right)^{-3/4} q^{-1/2}f^{1/2}g^{-1/2}\left(1-\frac{l^2 s}{s+\frac{1}{2}}\right)^{1/2}$$

(31)

To study the behaviour of the disk's temperature in the optically thick accretion disk, we can use Eq. (16) .It can be written as [27, 28]

$$T = 10^5\left(\frac{M_\odot}{M}\right)^{1/4} \dot{m}^{1/4}\left(\frac{R}{R_G}\right)^{-3/4} f^{1/4}g^{-1/4}\left(1-\frac{l^2 s}{s+\frac{1}{2}}\right)^{1/4}$$

(32)

4. Numerical results

4.1. The thermal equilibrium solutions

At high accretion rates, the disk is optically thick, radiation cooling is very effective (Q+=Qrad), the disk is geometrically thin and relatively cool, and outflows are weak or absent. However, at low accretion rates, the disk becomes optically thin, radiation cooling fails, matter stores heat, Qadv dominates, and the disk expands geometrically to become bulging and extremely hot (ADAf dominates).

To study this behavior, we will examine the thermal stability curve (S-Curve/Horseshoe Loop), the most well-known curve in accumulator disk physics. It relates the accretion rate on the vertical axis to the disk's surface density or temperature on the horizontal axis (in a logarithmic representation, $\log_{10}$). It's important to remember that optical depth depends on surface density and opacity. Through the graph we will see how the values of energy and cooling sources change as we move away from or closer to the black hole, and where truncation occurs where the flow changes from radiation-controlled to convection-controlled.

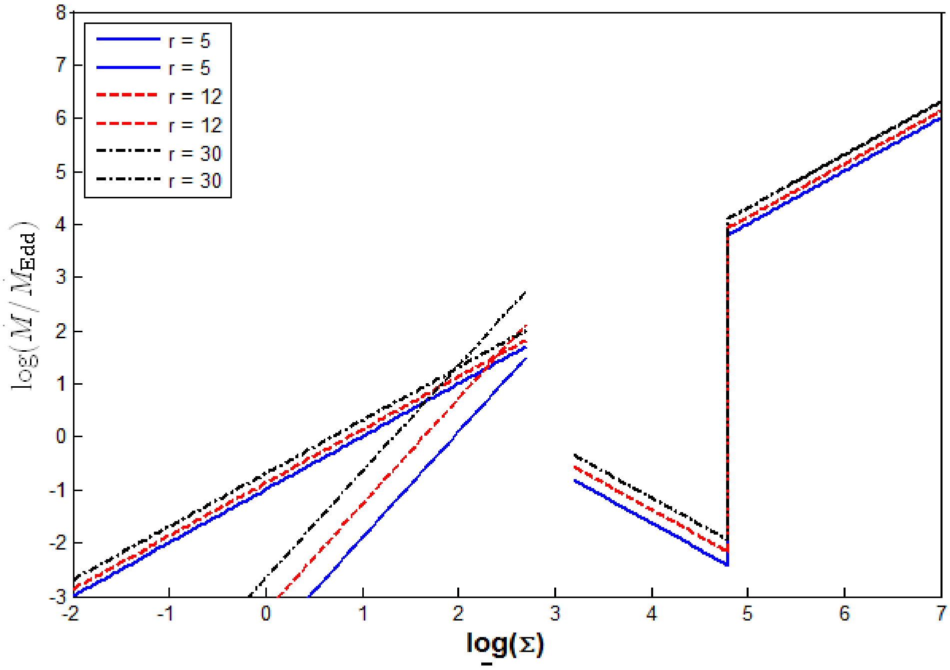

Fig.1. The thermal and viscous stability curve of the accumulator disk at three different radii in Schwarzschild radius units, the three curves (r=5, blue; r=12, red and r=30, black) for $\alpha = 0.1$

Through this curve we will discuss the apparent physical behavior based on the two optical states (thin and thick).

First: Left-hand side - Optically Thin Regime

These curves extend in the left-hand region where surface densities are low.

We observe two lines for each radius. This branching reflects different solutions to the energy equation in the optically thin regime. The upper part represents the ADAF solution, where photons cannot escape efficiently due to the low density. The thermal energy is trapped within the material and is adjected towards the black hole. The lower part represents the Radiative Cooling Dominated solution: if radiation fails to cool the disk, the energy is converted to convection.

Regarding the behavior of these curves with changing radii, we notice that as we move away from the black hole, from r=5 in blue to r=30 in dotted black, the curve rises upwards on the vertical axis. This means that at larger radii (r=30), the material requires a higher accumulation rate to achieve the same local surface density compared to very close regions (r=5), since orbital velocity and viscous forces decrease with distance from the center. As can be seen in the graph, the portion representing the radiative solution exceeds the portion representing the convective solution at larger radii.

We observe a gap representing an unstable phase, where the gas transitions from a very hot thin state (where the gas is fully ionized and plasma physics dominates) to a cold, thick optical state. There are no stable disk equilibrium solutions in this gap; therefore, the system jumps directly across it (which explains the occurrence of radioactive bursts, or outbursts, in X-ray diodes).

Second: Right Side - The Optically Thick Regime

This is divided into two very distinct parts separated by a sharp drop. The negative-slope branch, according to the laws of disk thermodynamics, means a region of thermal and viscous instability. Any small disturbance that increases the accumulation rate will lead to excessive heating, causing the disk to expand and jump energetically. This is what is known as the Standard Thin Disk model. Here, the disk is optically thick, but the dominant pressure is radiation pressure. Cooling occurs through photon diffusion. In this regime, the mechanism of thermal and viscous instability, responsible for forming the S-curve, becomes prominent. This sharp vertical break represents a state change point. As the surface density increases and the material cools, the opacity drops very sharply, altering the disk's ability to radiate cooling and forcing it to rapidly drop into a new, stable branch. The right-hand upward branch: here we move to a region where gas pressure dominates instead of radiation pressure. The slope here is perfectly positive, meaning this branch is thermally stable. The disk here is dense and radiates like a perfect black body. It is called the slim disk.

The effect of the radius on all curves: Considering the vertical arrangement of the three curves (r=5, blue at the bottom; r=12, red in the middle; r=30, black at the top), we find that the physical behavior is identical in shape but directly offset. The inner regions (r=5), closer to the black hole, have a deeper gravitational well and therefore produce much greater energy and viscous friction. This enables them to maintain their radiation stability at lower accumulation rates compared to the outer regions (r=30).

The ultimate conclusion of this complex physical behavior can be formulated in the following points: Accumulation disks do not operate at a uniform pace; rather, based on their density and feeding rate, they are divided into two completely separate worlds separated by an instability gap. One is an optically thin and hot world (left), dominated by ADAF (advanced external cooling) or pure radiative cooling, where the gas is unable to trap photons. The other is an optically thick and cold world (right), representing conventional (Shakura-Sunyaev) disks that radiate like a blackbody, and whose stability is controlled by the nature of the dominant pressure (stabilized gas pressure).

The negatively sloping curves (both the mid-branch on the thick side and the sharp breaks) reveal that the system undergoes dynamically unstable phases. When the radiation pressure becomes dominant or the ionization state of the gas (hydrogen) changes, the disk is unable to find a stable thermal equilibrium point. This oscillatory behavior (negative slope) is directly responsible for the phenomenon of outbursts, where the system is forced to abruptly jump between hot and cold states to dissipate matter and energy.

4.2. The temperature properties

The temperature of the accretion disk is key to understanding the spectrum of a black hole. It varies dramatically depending on the system's radiative efficiency, which is closely related to the optically thick and thin states. In the thick state, the temperature is relatively low compared to other systems and increases as we approach the event horizon. Because the disk is optically thick, photons generated by the viscosity collide with gas atoms multiple times before escaping, effectively cooling the gas. Temperatures often fall within the ultraviolet (UV) or soft X-ray range of stellar-mass black holes. The optically thin state, on the other hand, indicates that the matter is so low in density that photons escape almost immediately upon generation without interacting with the matter. The temperature rises to very high levels, and due to the lack of effective cooling, the thermal energy remains trapped in the ions. Hard X-rays and gamma rays are emitted from these regions.

In this section, we will discuss the physical differences in temperature between accretion disks that lose heat through radiation and those that retain it through advection, and what these differences reflect about the behavior of matter around a black hole.

First, we will consider the thin optical state. Then, we will discuss the behaviour of the curves that describe this state.

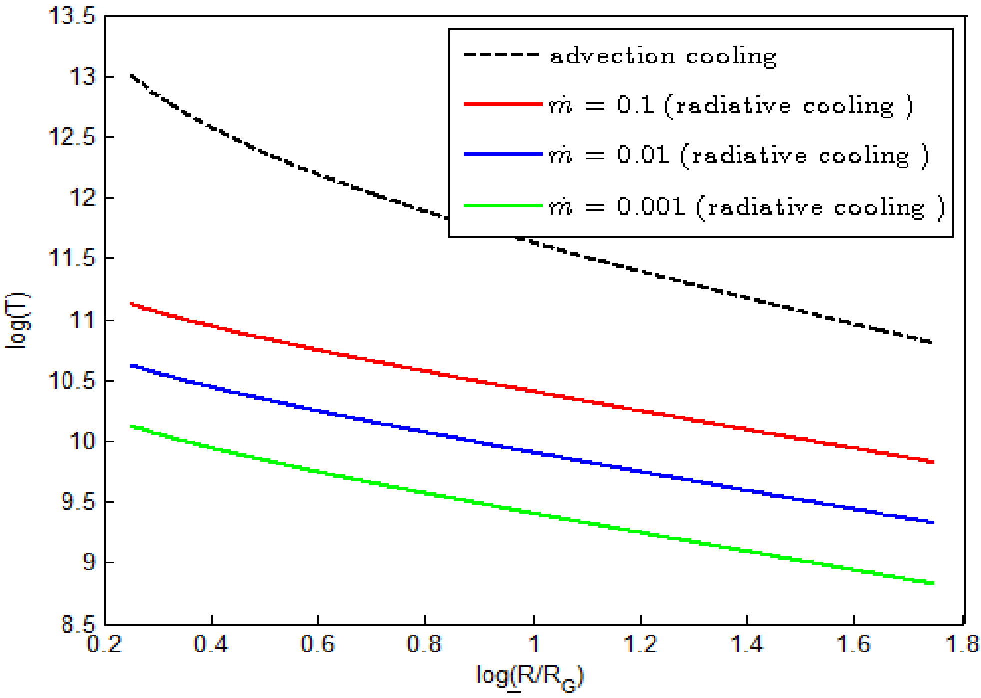

Fig.2. The radial temperature of optically thin accretion disks for $\alpha = 0.1$, with $l = 1$ and $s = 0.23$ for advection dominated solutions. However, for local radiative cooling dominated solutions, $s = 0.003$

The dashed curve, representing advection cooling, is the extreme case of inefficient cooling, as seen in ADAF models. The curve starts at very high values (extreme temperatures) where all the gravitational potential energy is converted into heat without dissipation.

The colored curves, corresponding to radiative cooling, are significantly lower than the advection curve. This means that radiation absorbs and releases heat from the gas, keeping it much cooler than in advection.

Regarding the effect of the accumulation rate, a higher accumulation rate leads to a higher temperature. This is because increased matter means more collisions between particles, generating greater viscous heat. Although radiative cooling is effective, the amount of energy generated is so large that it raises the overall temperature of the gas.

We conclude that in advection cooling, the gas fails to cool and remains swollen and extremely hot. In the case of radiative cooling: the gas "succeeds" in getting rid of the energy, so it remains cooler (relatively).

In the second case, we will also discuss the behaviour of the curves that describe the thick optical state.

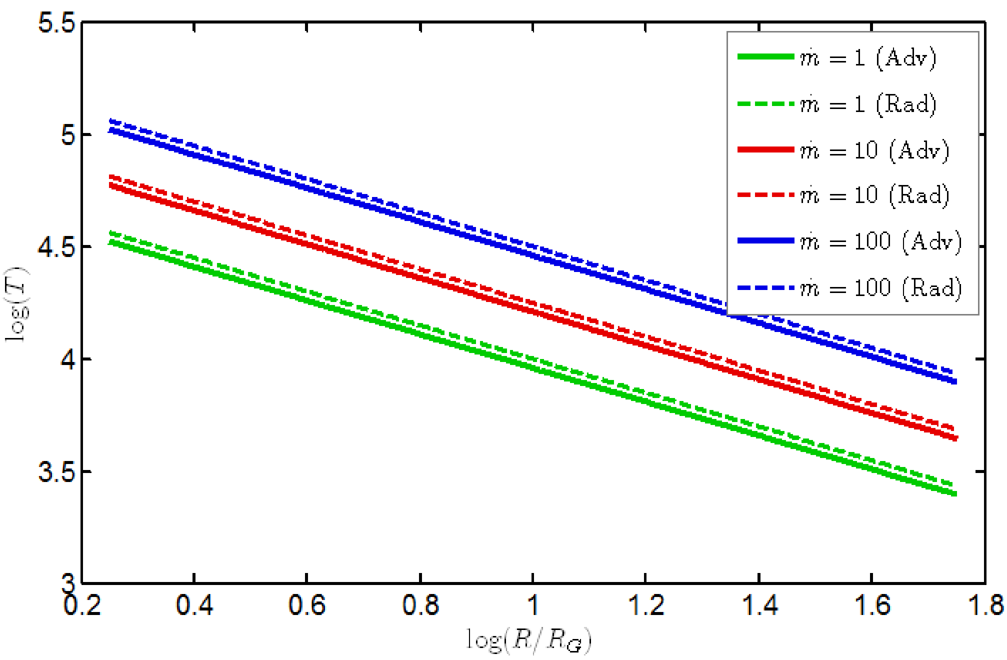

Fig.3. The radial temperature of optically thick accretion disks for $M = 10M_{\odot}$ and $\alpha = 0.1$, with $l = 1$ and $s = 0.23$ for advection dominated solutions. However, for local radiative cooling dominated solutions, $s = 0.003$

This graph represents the other end of the spectrum, the optically thick state, where the material is very dense. Here, we observe behavior that is completely different from the previous graph of the thin state.

We can see that the reason for this physical difference is due to;

The first thing that catches the eye in this graph is that the continuous (advection) and (radiative) curves for each accumulation rate are almost identical or very close. The physical explanation for this is that in the optically thick state, the material and radiation are in a state of tight thermal equilibrium. Photons are trapped within the material and constantly collide with it before escaping. This means that the gas temperature is "governed" by the system's ability to radiate as a blackbody. The tiny difference you see indicates that radiative cooling is so effective that it determines the temperature trajectory, whether or not convection is taken into account.

On the other hand, we clearly observe that the blue curve is higher than the red, and the red is higher than the green. This means that as the amount of accumulated material increases, the friction and viscosity increase, and consequently, the energy flux escaping from the surface increases, raising the "blackbody" temperature of the disk.

The final conclusion can be summarized by comparing the two statements illustrating each case

In the thin state (Figure 2): The system is radiatively inefficient. The large gap between the advection curve and the radiative cooling curves tells us that the matter prefers to retain heat and carry it towards the black hole rather than radiate it. Therefore, the temperatures are high.

In the thick state (Figure 3): The system is radiatively very efficient. The matching curves mean that the matter releases its energy almost immediately upon generation. Radiation is the absolute ruler here, so the temperatures remain modest.

In the thin state: The matter acts as a "battery," storing kinetic energy as enormous heat in the ions. This leads to the formation of a very hot "corona" around the black hole that emits hard X-rays.

In the thick state: The matter acts as a "radiator." Once the matter heats up slightly, it radiates intensely like a blackbody, which keeps the disk stable and relatively cool. This explains the appearance of soft X-rays or ultraviolet radiation from stable disks.

5. Conclusions and Discussion

This study demonstrates that the radius effect tells us that the proximity of matter to the event horizon alters the conditions of the thermal interaction. The overwhelming gravity in the inner regions compensates for the lack of matter, allowing the disk to maintain its thermal and radiative equilibrium at much lower accumulation rates compared to the outer regions, which require a massive injection of matter to achieve the same surface density. Another important finding is that the accumulation rate and optical depth are the determinants of the observed “personality” of a black hole; If matter is scarce (optically thin), the black hole appears "dim" in radiation but has a hot, pulsating flux. If matter is abundant (optically thick), the black hole appears very "bright" with a thin, orderly disk.

In conclusion, the results obtained in this study indicate that the thermodynamics of a black hole is not a single state, but rather a continuous struggle between viscosity (which raises the temperature) and the optical properties of matter (which determine whether this heat escapes as photons or falls into the abyss).We believe that this comparison supports the hypothesis of a "transition point" at which the thick disk disappears and is replaced by the thin flow.

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